BELLCOMM, INC.
1100 Seventeenth Street, N.W. Washington, D.C. 20036

SUBJECT: Feasibility of A Simplified LM Ascent and Rendezvous Scheme - Case 310

DATE: March 26, 1968

FROM: D. R. Anselmo
D. J. Toms

ABSTRACT

This paper presents a simplified ascent and rendezvous scheme designed to minimize LGC flight software requirements. The scheme includes simplified launch-to-insertion guidance and a concentric flight plan re-shaped to allow more complete ground support.

The simplified launch to insertion guidance has been simulated and preliminary performance and dispersion data is presented. This data shows that acceptable insertion conditions are attained with no appreciable ΔV penalty. The flight profile is discussed maneuver by maneuver and the necessary equations for on-board targeting are developed.

A list of programs for nominal and backup use is provided. Based on estimates of the software required to implement this ascent and rendezvous scheme, it appears that the LGC requirements for these portions of the profile can be reduced by about seventy-five percent.

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MEMORANDUM FOR FILE

INTRODUCTION

This study was performed as part of a Bellcomm effort to determine the feasibility of developing simplified LGC flight software while meeting the minimum requirements for carrying out a lunar landing mission. The study's objective was to determine the feasibility of carrying out a lunar mission using half the fixed and erasable memory of the LGC. The portion of the mission examined here begins at LM launch and extends through rendezvous.

The scheme examined in this report achieves considerable LM software simplification by introducing two modifications to the basic concentric flight plan:

1. Simplified launch-to-insertion guidance is employed.

2. The CSI maneuver is delayed until after a circularization maneuver is performed.

The first modification reduces computer word requirements by eliminating the present powered ascent guidance equations and using an open loop pitch program in conjunction with the cross product steering programs which are contained in the LGC under the existing system. It is shown that this simplified guidance scheme produces acceptable dispersions and no appreciable ΔV penalty.

Performing the CSI maneuver on a circular orbit has the effect of permitting all on-board calculations to be done with very simple equations and routines. In addition, the delayed CSI occurs on the front side of the moon and hence this maneuver (indeed, all maneuvers through TPI) can be targeted from the ground. Much of the on-board computational capability, therefore, has to be carried only for backup purposes. Delaying CSI also has the effect of extending the timeline thereby permitting more flexible mission planning and reducing the work load for the crew. The total time for launch to terminal braking is approximately 4.5 hours.

The body of this report is divided into three sections: Profile, Profile Calculations, and Performance and Dispersions. The Profile section is a general description of the flight profile including a discussion of lighting and tracking constraints. The Profile Calculations section describes each maneuver and presents the appropriate equations. The Performance and Dispersions section contains the results of simulated runs showing dispersions at insertion, ΔV costs and a discussion of other dispersions in the profile. The report also has appendices which define the LGC requirements for the profile studied and the simplified guidance equations for launch-to-insertion.

PROFILE

The LM ascent profile is shown in Figure 1 and covers the period from LM liftoff to terminal braking. For a nominal mission the earliest possible and latest possible times for liftoff are supplied from the ground. After liftoff the LM rises vertically for a fixed time (a time of 12 seconds was used for the simulated runs discussed in this report). At the end of the vertical rise phase the LM pitches at a high rate for a specified time. Thereafter the LM pitches at a low rate until a specified time at which the cross product steering program is switched on. The remainder of powered ascent is guided closed-loop to the proper cutoff conditions. Cutoff occurs at approximately 400 seconds after launch and results in an orbit with an apolune of 40 nautical miles (this altitude for apolune was chosen in order to insure that the CSM would always be within rendezvous radar range long before the first circularization maneuver). The CDH1 maneuver establishes a circular orbit thereby creating conditions very favorable for the subsequent CSI maneuver. Since CSI is performed on a circular orbit, the direction of the CSI burn is always along the velocity vector as well as along the local horizontal so the burn is the most efficient possible in terms of ΔV. The circular orbit also allows CSI to be delayed until lighting and tracking conditions are the most favorable.

Specific equations for on-board calculation of the CDH and CSI maneuvers are given in the Profile Calculations section of this report. These equations are developed to illustrate the simplicity of the calculations involved; it is felt by the authors that these calculations could be performed by the use of suitable charts prepared pre-flight.

In this report CSI and TPI are placed at the same longitude (over the landing site) since the most favorable lighting and tracking conditions for TPI are also very favorable for CSI. The placement of TPI was guided by the following identified constraints.

1. A requirement for 15 minutes of LM to CSM visibility prior to TPI with the line of sight 30° clear of the sun line.[1*]

2. Optical tracking requirements as follows:[2]

   (a) Continuous CSM (optical) tracking of the LM from approximately 35 minutes before TPI until 15 minutes before TPI.

   (b) Continuous CSM (optical) tracking of the LM from 5 minutes after TPI through the last terminal midcourse correction, approximately 30 minutes after TPI.

   (c) The LM must not be separated from the CSM by more than 10 n.mi. when the undocked vehicles enter the region of a lighted lunar background at the end of the terminal transfer.

3. The time between MSFN tracking acquisition and execution of a maneuver should be about 20 minutes. This allows approximately 10 minutes for tracking, 2 to 3 minutes for up link of data and approximately 7 minutes on-board for verification and pre-thrust maneuvering.[3]

The first of these constraints can be satisfied by placing TPI no sooner than 32 minutes before the CSM enters darkness. The optical constraints can be satisfied by placing TPI between 5 and 35 minutes prior to CSM entrance into darkness. The last constraint can be satisfied by placing the CSM's TPI position between 50° east and 50° west of the earth-moon line.

If the landing sites between 42°E longitude and 42°W longitude are considered all of the above constraints can be satisfied by placing TPI over the landing site. With CSI also placed over the landing site the launch window and the nominal times for all maneuvers are the same for all landing sites.

PROFILE CALCULATIONS

The powered ascent from launch to insertion.

The present LM guidance equations used from LM lunar launch to orbit insertion are based on an explicit guidance scheme which controls insertion altitude and the insertion velocity vector.

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*While this constraint originated with optical tracker considerations, it is assumed that astronaut verification of CSM elevation at TPI will also require a line of sight clear of the sun.

The scheme presented in this report consists of an open loop powered ascent up to a fixed time prior to nominal orbit insertion followed by closed loop guidance for the remainder of powered ascent. A vertical rise for a specified time, a pitch over at a high pitch rate for a specified time, followed by a low pitch rate constitutes the open loop portion of the profile. The five constants involved (vertical rise time, high pitch rate, high pitch rate time, slow pitch rate, and guidance switch-over time) are based on a nominal pre-targeted trajectory. The guidance mode employed is the well known $\overline{V}_G x \dot{\overline{V}}_G = 0$ (cross product steering) explicit scheme.

The cross product equations used involve the derivative of the required velocity equation ($\dot{\overline{VR}}$). This derivative can be either analytically evaluated or approximated by first differences. The need for this derivative can be eliminated if the $\overline{V}_G x \overline{A}_t = 0$ law is employed rather than $\overline{V}_G x \dot{\overline{V}}_G = 0$ law. The simpler alternative ($\overline{V}_G x \overline{A}_t = 0$) was studied and was found to be problematical in that a large discontinuity in the commanded thrust direction occurs at guidance switch-over. In addition, neglecting the $\dot{\overline{V}}_r$ and $\overline{g}$ accelerations would result in lower insertion altitudes. Calculation of the derivative of $\overline{V}_r$ results in an improved closed-loop system at the cost of the computer memory required to make the calculation.

The specific on-board equations required for this powered ascent mode are developed in Appendix II.

<u>The first constant delta height maneuver (CDH1)</u>

CDH1 is a circularization maneuver performed at apolune of the insertion orbit. For a nominal insertion this maneuver occurs approximately 56 minutes after SI. The required time and $\Delta\overline{V}$ for this maneuver can be computed on-board using the following calculations:

$$a = \frac{\mu R}{2\mu - RV^2} \tag{1}$$

$$t_{CDH1} = t_{SI} + \pi \sqrt{\mu/a^3} \tag{2}$$

t_{CDH1} = time at which CDH1 should occur.

t_{SI} = time at which SI occured.

$\overline{R}$, $\overline{V}$ = any position and velocity attained after SI (this can be an updated state vector obtained from the rendezvous radar).

$$\Delta\overline{V} = \left(\sqrt{\mu/RA} - VA \right) \hat{U} \tag{3}$$

RA, VA = radius and velocity at apolune (obtained with KEPLER routine)

$\hat{U}$ = unit local horizontal vector at flight azimuth.*

For central and eastern landing sites it is expected that ground tracking and communication time will be sufficient to permit the required time and $\Delta\overline{V}$ to be supplied from the ground. For far western landing sites (42°W) the time between SI and loss of MSFN is approximately 10 minutes. Since this may prove to be insufficient time for a ground based computation the maneuver may have to be computed on-board in this case. Tabular data may, however, be sufficient for this calculation.

The Concentric Sequence Initiation (CSI)

With this profile the CSI maneuver is the initiation of a Hohmann transfer. This maneuver occurs on the front side of the moon and is placed so that sufficient time is available for ground tracking, ground computation, and data up-linking both before and after the maneuver. In this study CSI is placed so as to occur over the landing site. As mentioned earlier, placing CSI over the landing site satisfies lighting and tracking constraints for all landing sites and for all anticipated lighting conditions; it does not, however, maximize the duration of the launch window.

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*The ^ appearing over a quantity is used to designate a unit vector.

If loss of ground tracking or ground communication occurs, the time for this maneuver and the required $\Delta\overline{V}$ can be computed on-board using the following method.

First we consider the calculation of CSI time after the first CDH maneuver has been executed. Referring to Figure 2, the angle ϕ between the LM present position and the position at TPI is calculated. First a unit vector in the direction of the CM's TPI position is projected into the LM's flight plane.

$$P\hat{R}TPI = \overline{NRTPI} \times \overline{H}$$

where $$N\hat{R}TPI = \overline{H} \times \overline{RTPI}$$

$\overline{H}$ is the angular nomenturn vector of the LM orbit i.e., $(\overline{R}x\overline{V})$.

$\overline{RTPI}$ is the position vector of the CM at TPI.

Then ϕ is given by

$$\phi = [1-(S)1]\,[1-(D)1]\pi + S\cos^{-1}[\hat{R}\cdot PR\hat{T}PI] \tag{4}$$

where: $S = \mathrm{Sign}\,[\overline{H}\cdot(\overline{R}xPR\hat{T}PI)]$

$D = \mathrm{Sign}\,[\overline{R}\cdot P\hat{R}TPI]$.

The CM's lead angle at TPI is given by

$$\Delta\phi = \frac{\Delta h \cot EL}{R} \tag{5}$$

where: Δh = differential height between LM and CM orbits at TPI

EL = elevation of CM above LM local horizon at TPI

R = LM orbital radius.

The total coast angle from present position to the LM position at TPI is then

$$A = 2\pi + \phi - \Delta\phi . \tag{6}$$

The time for CSI execution is

$$t_{CSI} = T \left[1 + \frac{W_2}{W_1 - W_2}\right] + \frac{A - \pi - W_2(t_{TPI} - t_{HOH})}{W_1 - W_2} \tag{7}$$

where

W_1 = orbital rate at CDH1 altitude = $\sqrt{\frac{\mu}{RP^3}}$

W_2 = orbital rate at CDH2 altitude = $\sqrt{\frac{\mu}{RA^3}}$

T = present time

$t_{HOH} = \pi \sqrt{\frac{(RP + RA)^3}{8\mu}}$

t_{TPI} = time of TPI supplied from ground or chart

RP = perilune of transfer from CSI to CDH2

RA = apolune of transfer from CSI to CDH2.

The $\Delta\overline{V}$ at CSI can be computed using

$$\Delta\overline{V} = \left[\sqrt{\frac{\mu\ RA}{(RA + RP)\ RP}}\ \hat{U}\right] - \overline{V} \tag{8}$$

where

$\overline{V}$ = LM velocity at time of CSI (obtained with Kepler routine).

The second constant delta height maneuver (CDH2)

This maneuver is the same as CDH1 except that for all nominal missions the computations are done on the ground.

The terminal phase initiation maneuver (TPI)

A nominal time for TPI is given to the LM before launch. This nominal time is used to target the CSI maneuver. Because of dispersions in the CSI and CDH2 maneuver the actual time of TPI will differ somewhat from the nominal time. Ordinarily, the actual time of TPI and the $\Delta\overline{V}$ for the maneuver are computed on the ground. In the case of ground tracking or communication failure the actual time of TPI can be obtained on-board as follows:

Propogate both vehicles to the nominal time of TPI using the KEPLER routine.

Calculate the central angle between the LM position and its desired TPI position.

$$A' = A - 2\pi \tag{9}$$

where A is calculated using Equation (6).

Obtain a new estimate for t_{TPI}:

$$t_{TPI} = t_{TPI} + A' (W_1 - W_2) \tag{10}$$

where W_1 = orbital angular rate of the LM

W_2 = orbital angular rate of the CSM.

By repeating the above process using the new estimate as an input the time of TPI can be obtained to any desired degree of accuracy.

The $\Delta\overline{V}$ for the maneuver is obtained as follows:

Propagate the LM to the time of TPI using the KEPLER routine (position $\overline{R1}$).

Propagate the CSM to the time of rendezvous (position $\overline{R2}$) using the KEPLER routine (the time between TPI and rendezvous is set prior to the mission and is independent of Δh).

Obtain the desired LM velocity vector at TPI ($\overline{VTPI}$) using the TIME-THETA routine iteratively to determine P in conjunction with the following equation.[4]

$$\overline{VTPI} = \frac{\sqrt{\mu P}}{R1\ R2\ \mathrm{Sin}\theta} \left[\overline{R2} - \left[1 - \frac{R2}{P} (1-\cos\theta) \right] \overline{R1} \right] \tag{11}$$

where: θ = the angle between R1 and R2

P = semi-latus rectum of transfer ellipse

$$\text{Then } \Delta\overline{V} = \overline{VTPI} - \overline{V}. \tag{12}$$

Midcourse Corrections

With TPI over the landing site the time between TPI and loss of MSFN is approximately 23 minutes for a 42°W site and much longer for central and eastern sites. This should be ample time for ground computation of a midcourse correction. If for some reason a midcourse correction is desired and the ground cannot supply it (for example, a second mid-course correction) the computation can be done on-board using the technique used for determining $\overline{VTPI}$.

PERFORMANCE AND DISPERSIONS

Powered Ascent

A preliminary determination of the validity of powered ascent using an open loop pitch profile followed by cross product steering was performed by testing the behavior of these equations for both nominal and perturbed cases. For this preliminary study, perturbations were considered singly, no correlation effects were considered. The results show that it is possible to achieve cut off conditions which meet the standard insertion target conditions at apolune very precisely. The guidance scheme does not control altitude at standard insertion. For this reason the open loop portion of the ascent profile must be designed such that a safe insertion altitude is assured in the presence of worst case initial conditions (worst case vehicle performance characteristics, and worst case error sources). The simulations performed indicate that safe insertion conditions will be obtained for a wide set of perturbations. The resulting errors introduced at the target point (apolune) can be absorbed in later maneuvers.

Two different guidance switch-over times were examined, one 200 seconds after liftoff (approximately halfway through the burn), and one 386 seconds after liftoff (approximately 14 seconds prior to nominal cutoff). In both cases safe insertion conditions resulted.

Switch-over at the earlier time produces more accurate target conditions but generally results in lower insertion altitude. Late switch-over results in a wider dispersion in insertion altitude and slightly less accuracy in insertion velocity direction.

The results of these simulations are given in detail in Table I. Results obtained for open loop flight to cutoff are also given. The cutoff in this case was determined by the attainment of a preset inertial velocity magnitude. One open loop flight (3σ low thrust) resulted in unsafe insertion conditions.

The 200 second switch-over resulted in a maximum dispersion in apolune time of arrival of approximately $\pm$ 20 seconds, the particular error source in this case being 3σ high and low thrust. The flight plan considered can handle this dispersion within the planned range of differential altitudes. The dispersion in apolune arrival time can be equated to dispersions in lift-off time, hence a 40 second range in apolune arrival time results in a reduction of 40 seconds in the available lunar launch window.

The ΔV costs associated with this powered ascent scheme for the nominal and perturbed cases are given in Table I. The present budget allows 6030 feet per second for launch into the nominal 10 n.mi. by 30 n.mi. orbit with an additional 30 fps budgeted for flexibility and dispersions. The trajectories in Table I employ a nominal 8.25 n.mi. by 40 n.mi. insertion. The standard open loop (unperturbed) target run to insertion required 6010 feet per second. Employing a 200 second switch-over time required 5998 feet per second and resulted in a 6.3 n.mi. by 40 n.mi. orbit. Guidance switch-over at 386.5 seconds does not produce a significantly different ΔV cost than the 200 second switch-over.

Minimum Δh

In order to obtain the longest possible launch window it is desirable to use the widest possible range of Δh. Maximum Δh is restricted, of course, to be less than or equal to the differential height at CDH1 unless a retrograde maneuver is permitted at CSI. Minimum Δh is restricted by the amount of dispersion that can be tolerated in the time of TPI. The dispersion in TPI time grows rapidly as Δh is reduced below 10 nautical miles. This is shown in Figure 3 which is a graph of the following equation.

$$\sigma_t = \frac{\sigma_A}{W_1 - W_2}$$

where σ_t = dispersion in TPI time

σ_A = dispersion in CSM lead angle at time of nominal TPI

W_1 = orbital angular rate of the LM

W_2 = orbital angular rate of the CSM

In this report a minimum Δh of 10 n.mi. and a maximum Δh of 20 n.mi. was used for all calculations. This results in a launch window duration of 80 seconds. As mentioned earlier the dispersion in the time of CDH1 can be as great as ± 20 seconds (for 3σ high and low thrust). This reduces the launch window to an effective length of 40 seconds.

CONCLUSIONS

It is concluded that considerable LGC software simplification can be achieved by employing one or both of the modifications presented here, namely the simplification of powered ascent guidance and the simplification of on-board rendezvous calculations made possible by greater reliance on ground support. The introduction of a delayed CSI provides additional MSFN coverage to support the use of these rendezvous calculations. In addition it may be possible to perform the CSI and CDH calculations described by means of graphical or tabulated data. It should be emphasized that the two modifications are independent; each can be used alone and each leads to a significant reduction in software.

It is shown that this software simplification can be achieved with nearly the same performance requirements while meeting presently identified constraints. Further validation of dispersions, safety, and mission success probabilities would have to be performed before this scheme could be considered for actual use.

Based on the specific equations and present LGC programs retained for the implementation of this LM ascent profile it is estimated (Appendix I) that a seventy-five percent reduction in LGC software requirements associated with LM ascent and rendezvous could be achieved.

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Attachments:
- References
- Appendix I
- Appendix II
- Table I
- Figures 1, 2, 3

REFERENCES

1. "Introduction of the Second Design Reference Mission", M. V. Jenkins, August 24, 1965.

2. Merritt, A. C., "Revisions to the Lunar Landing Mission Rendezvous", Bellcomm Memorandum for File, February 29, 1968.

3. Personal communication with J. D. Alexander, Mission Planning and Analysis Division, MSC, February 20, 1968.

4. Battin, R. H., Astronautical Guidance, McGraw Hill Book Company, 1964.

APPENDIX I

PROGRAMS

The flight software required for this ascent and rendezvous profile can be conveniently divided between nominal and backup requirements. The following list summarizes the nominal LGC requirements in terms of present programs and specific equations which have been developed in the text.*

1. Pre-launch - Ground calculations.

2. Powered ascent.

   (a) Open loop Equations AII-1, AII-2, AII-3.

   (b) Guided flight Equations AII-4, AII-12.

   (c) Present cross product steering routine.

3. CDH maneuver

   (a) Present External ΔV program.

   (b) Present cross product steering routine.

   (c) Present conic Kepler routine.

   (d) Equations (1), (2), and (3).

4. CSI maneuver

   (a) Present External ΔV program.

   (b) Present cross product steering routine.

   (c) Ground calculations.

5. TPI maneuver

   (a) Present External ΔV program.

   (b) Present cross product steering routine.

   (c) Ground calculations.

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*Supporting routines such as Middle gimbal and Servicer have not been considered in this list but are included in the LGC word estimates given at the end of this appendix.

6. Midcourse maneuvers

   (a) Present External ΔV program.

   (b) Present conic Time-theta and Kepler routines.

   (c) Present cross product steering routine.

   (d) Equation (11) and (12).

Suggested backup programs for the LGC which are additions to the nominal programs:

1. CSI maneuver - Equations (4), (5), (6), (7) and (8).

2. TPI maneuver - Equations (4), (5), (6), (9) and (10).

Present programs and routines which are employed for LM powered ascent and rendezvous maneuvers are given in the following lists. The MIT budget and the estimated word requirements are given.

PROGRAMS	PRESENT BUDGET	NEW ESTIMATE
AGS Initialization	100	100
Rendezvous Out of Plane Display	120	0
Preferred Tracking Attitude	50	50
External ΔV	150	50
Predicted Time of Launch	200	0
General Lambert	120	0
APS Abort	100	100
Thrust monitor	70	0
CSI Pre-thrust	70	0
CDH Pre-thrust	100	0
TPI Pre-thrust	460	0
TPM Pre-thrust	95	0
APS thrust	30	30

Basic Routines		
Conic	1050	800
Orbit Integration	1400	0
Latitude Longitude Altitude	170	0
Planetary Initial Orientation	280	0
Initial Velocity	75	0
Rendezvous Parameter	90	0
Middle Gimbal	75	0
Target Routines		
Predicted Launch Time (CGP)	650	0
CSI Initiation	440	130
Constant Delta Altitude	300	50
TPI Initiation	0	120
Predicted Launch Time (D.T.)	100	0
TPI Search	200	0
Powered Flight Routines		
Servicer	850	500
Cross Product Steering	75	60
VG calculation	100	0
Time of Burn Calculation	105	0
Ascent Guidance	700	100
Totals	8325	2090

It should be noted that while these estimates include on-board calculations of both the CDH and CSI maneuvers the possibility of using on-board charts for these maneuvers has not been excluded.

APPENDIX II

Powered Flight Equations:

The open loop portion of ascent is performed in three stages. The required acceleration vector direction is given by the following for the three open loop flight phases,

$$\hat{A}_t = \hat{R}_L \qquad \text{(vertical rise) } 0<T<T_V \qquad \text{AII-1}$$

$$\hat{A}_t = \hat{R}_L \cos W_h T - \hat{U}_L \sin W_h T$$

$$\text{(pitch over) } T_V<T<T_H \qquad \text{AII-2}$$

$$\hat{A}_t = \hat{R}_L \cos (W_L T + W_h T_H) - \hat{U}_L \sin (W_L T + W_h T_H)$$

$$\text{(low pitch rate) } T_H<T<T_{SO} \qquad \text{AII-3}$$

where

$\hat{R}_L$ = unit radius vector at liftoff

W_h = high pitch rate

W_L = low pitch rate

T_V = duration of vertical rise

T_H = T_V + duration of high pitch rate

T = elapsed time from liftoff

$\hat{U}_L$ = unit $(\hat{H} x \hat{R}_L)$, where $\hat{H}$ is unit angular momentum vector of desired orbit

T_{SO} = time of switch-over to closed loop guidance.

The guided portion of powered ascent is performed with the cross product steering equations presently employed in the on-board software. An additional calculation required is the development of the required instantaneous velocity ($\overline{V}_R$). The derivative of $\overline{V}_R$ with respect to time must also be computed. This can be done numerically by first differences or analytically. The $\overline{V}_R$ equation and the analytic $\dot{\overline{V}}_R$ equations are developed in what follows.

$$\overline{V}_R = \left[\frac{2\mu \ RA}{(RA + R)\ R} \right]^{1/2} \hat{U} \qquad \text{AII-4}$$

where

RA = apolune target radius

R = magnitude of present radius vector

μ = lunar gravitational constant

$\hat{U}$ = unit local horizontal ($\overline{H}x\overline{R}$)

The necessary equation for $\dot{\overline{V}}_R$ is now developed

$$\overline{V}_R = V_R \ e^{-j\omega t}$$

$$\text{where } \omega = \frac{V_{Tangential}}{R} = \frac{\overline{V} \cdot \hat{U}}{R} \qquad \text{AII-5}$$

$\overline{V}$ = actual velocity

Note that $e^{j\omega t}$ is the instantaneous local horizontal if we define

$$e^{-j\omega t_o} = \hat{U}_o . \qquad \text{AII-6}$$

Taking the derivative of $\overline{V}_R$ we obtain

$$\frac{d\overline{V}_R}{dt} = - V_R \, [j\omega e^{-j\omega t}] + \frac{dV_R}{dt} \, [e^{-j\omega t}] \qquad \text{AII-7}$$

where

$$\frac{dV_R}{dt} = \left[\frac{RA + 2R}{2R \, (R+RA)}\right] V_R \frac{dR}{dt} = K \frac{dR}{dt} \qquad \text{AII-8}$$

Hence

$$\frac{d\overline{V}_R}{dt} = K \frac{dR}{dt} \, [e^{-j\omega t}] - V_R \left[\frac{\overline{V} \cdot \hat{U}}{R}\right] je^{-j\omega t} \qquad \text{AII-9}$$

if we observe that:

$$\frac{dR}{dt} = \frac{\overline{V} \cdot \overline{R}}{R} \qquad \text{AII-10}$$

and

$$e^{-j\omega t} = \hat{U}$$

then

$$je^{-j\omega t} = \hat{R} \qquad \text{AII-11}$$

The required derivative is then given by

$$\frac{d\overline{V}_R}{dt} = K(\overline{V} \cdot \hat{R})\hat{U} - V_R\left(\frac{\overline{V} \cdot \hat{U}}{R}\right) \hat{R} \qquad \text{AII-12}$$

TABLE I

POWERED ASCENT SIMULATIONS

PERTURBED PARAMETER	OPEN LOOP POWERED ASCENT: BURN DURATION	ΔV	FLIGHT PATH ANGLE	ALTITUDE INSERTION	ALTITUDE PERICENTER	OPEN LOOP PLUS CROSS PRODUCT POWERED ASCENT: GUIDANCE SWITCH-OVER TIME	ALTITUDE AT INTT	ALTITUDE AT INS.	ALTITUDE PERICENTER	FLIGHT P. ANGLE	BURN DURATION	ΔV
STANDARD RUN ISP 306.3 SEC WGT 10,209 LBS APS THRUST 3,500 LBS LOW PITCH RATE -.1263 DEG/SEC DURATION H.P. RATE 6.6786 SEC	403.3 (SEC)	6010.4 (FPS)	-.0152 (DEG)	49,764.8 (FT)	49,752.5 (FT)	200 (SEC)	31,703.4 (FT)	38,149.8 (FT)	38,149.7	.0022 (DEG)	402.6 (SEC)	5,997.5 (FPS)
						386.5	49,706.6	49,769.2	49,769.2	.00036	403.2	6009.7
ISP 303.3 (3σ LOW)	402.3	6011.4	.0554	50,551.6	50,388.5	200	31,798.3	38,433.1	38,432.8	.0022	401.6	5997.3
						386.5	50,397.2	50,498.5	50,351.1	-.0032	402.2	6010.2
ISP 309.3 (3σ HIGH)	404.2	6009.4	-.0846	48,985.3	48,600.3	200	31,610.5	37,870.8	37,870.8	.0022	403.7	5997.7
						386.5	49,033.0	49,052.6	48,904.0	-.0035	404.2	6009.3
THRUST 3395 (3σ LOW)	413.6	5982.8	-1.1914	30,385.6	-36,015.5	200	27,408.8	28,632.8	28,632.5	.0019	414.4	5997.2
						386.5	33,157.2	31,970.7	31,900.4	.0377	414.8	6006.1
THRUST 3605 (3σ HIGH)	393.3	6034.9	1.1057	67,269.7	20,903.0	200	36,021.7	47,112.2	47,111.9	.0022	391.7	5999.9
						386.5	66,536.0	66,939.8	65,474.5	-.158	394.9	6069.0
WGT 10109	399.9	6018.9	.3643	55,794.7	49,302.0	200	33,138.6	41,193.1	41,192.8	.0025	398.9	5998.1
						386.5	55,280.9	55,534.2	55,494.6	-.0266	399.8	6017.0
WGT 10309	406.6	6001.8	-.3936	43,644.4	35,187.5	200	30,297.3	35,103.4	35,103.3	.0020	406.4	5997.1
						386.5	44,265.8	44,036.9	44,020.5	.0177	406.8	6006.7
DURATION OF H.P. RATE -.05	403.9	6023.3	.3407	55,852.7	50,155.0	200	32,895.5	40,864.1	40,863.9	.0024	402.9	6001.9
						386.5	55,207.0	55,531.0	55,511.6	-.0187	403.7	6020.3
DURATION OF H.P. RATE .05	402.6	5997.6	-.3707	43,700.0	36,164.0	200	30,506.9	35,430.0	35,429.8	.0020	402.4	5993.2
						386.5	44,187.0	44,002.0	44,977.1	.0218	402.9	6002.6
LOW PITCH RATE (-1%)	403.5	6016.1	.2444	52,657.7	49,597.4	200	31,951.2	38,874.3	38,874.1	.0023	402.7	5998.6
						386.5	52,192.3	52,430.5	52,421.4	-.0129	403.5	6014.4
LOW PITCH RATE (+1%)	403.0	6004.8	-.2691	46,938.7	33,026.7	200	31,460.2	37,439.0	31,438.9	.0022	402.6	5996.4
						386.5	47,266.5	47,160.6	47,152.1	.0126	403.1	6006.5

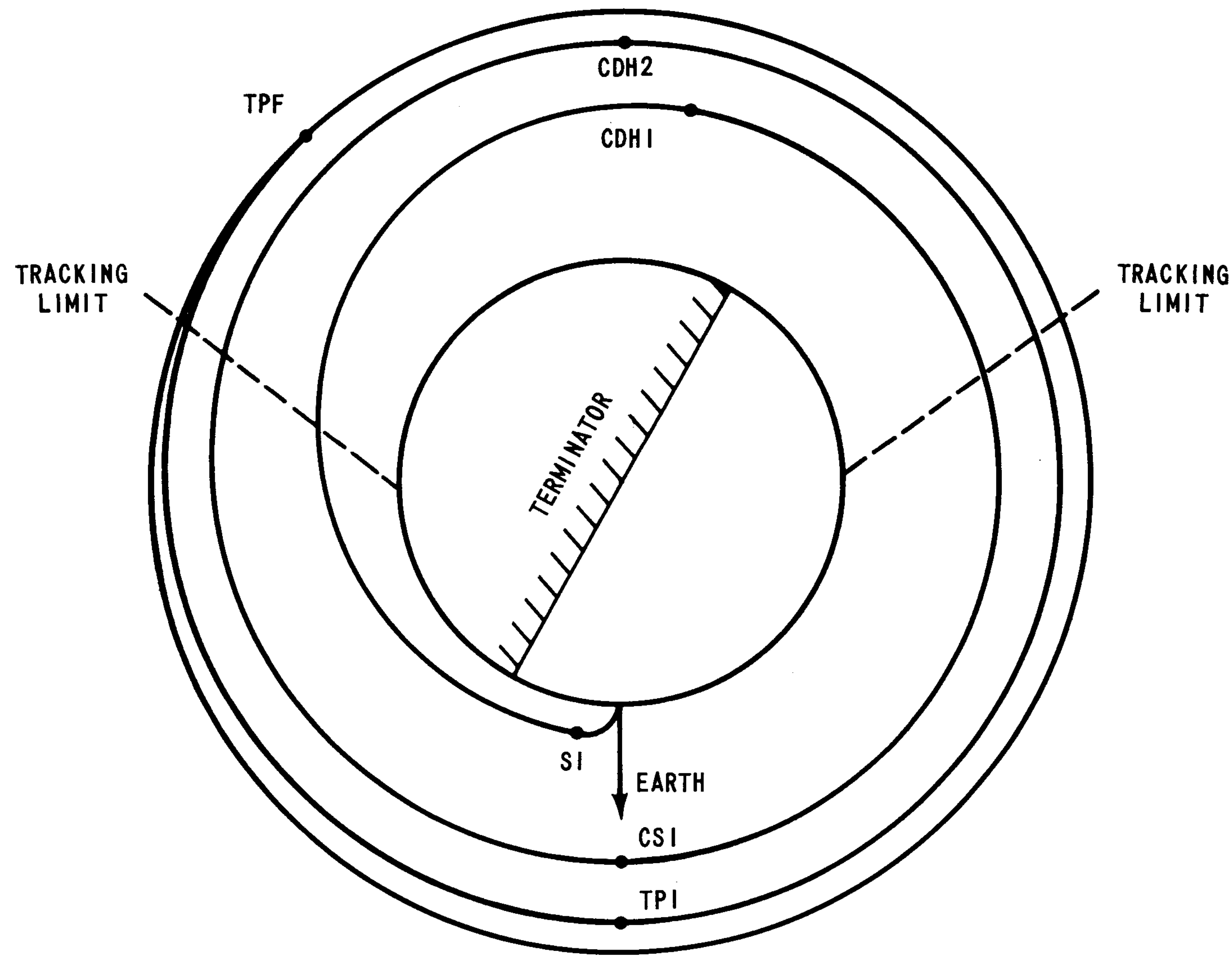

FIGURE 1 - CONCENTRIC FLIGHT PLAN WITH DELAYED CSI

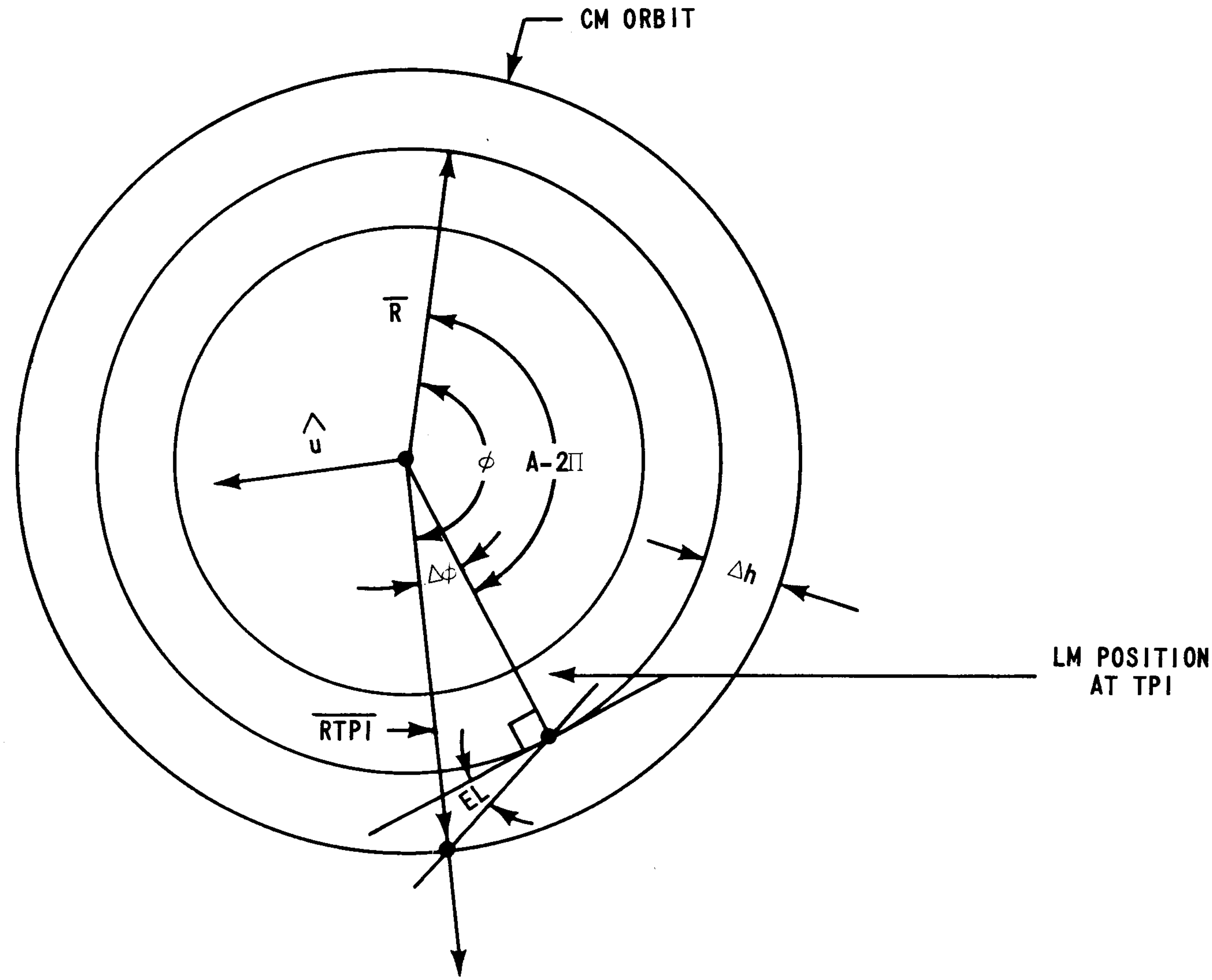

FIGURE 2 - TPI GEOMETRY

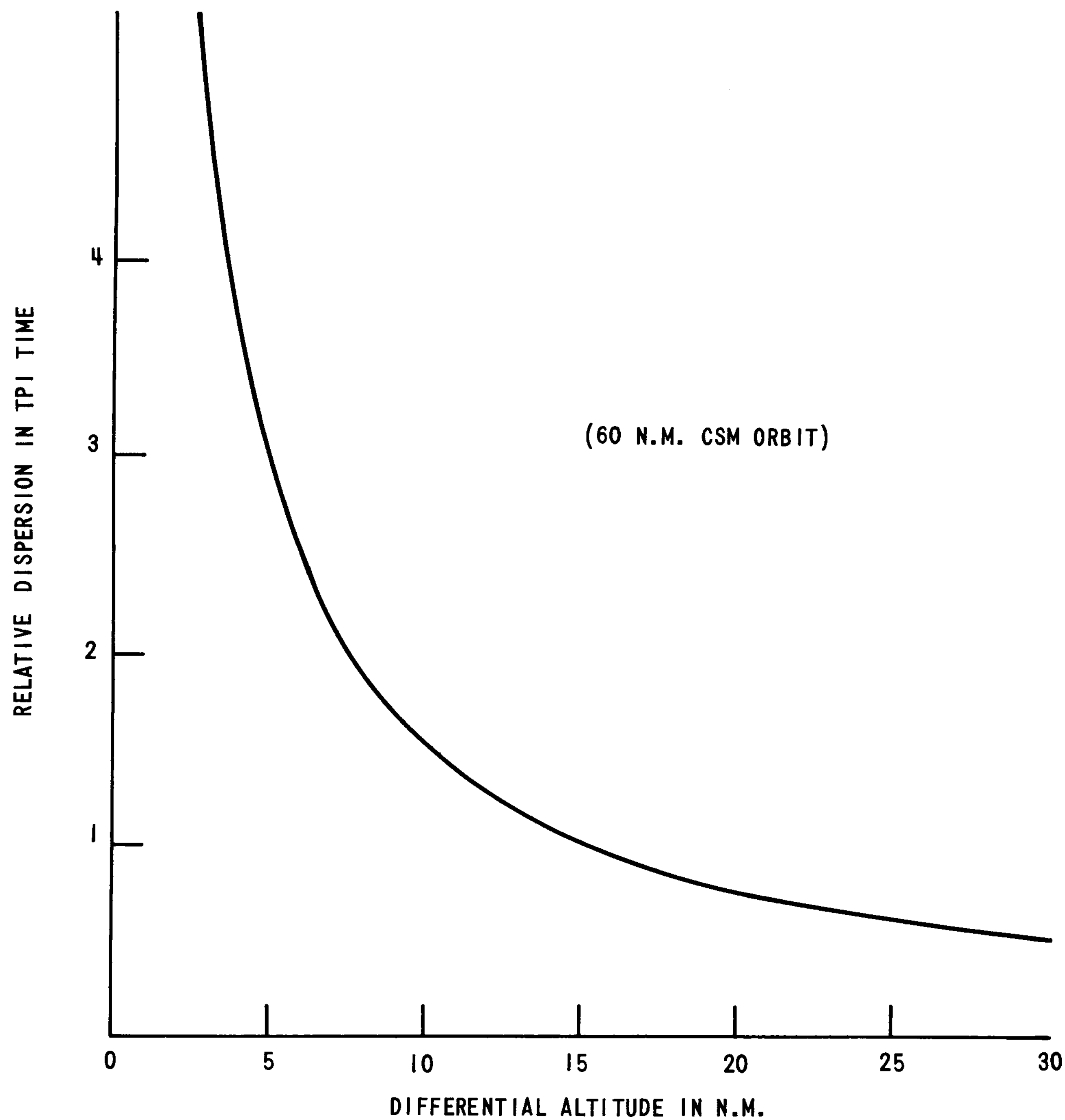

FIGURE 3 - RELATIVE DISPERSION IN TPI TIME AS A FUNCTION OF Δh

BELLCOMM, INC.

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